



2.1.1 Real finite-dimensional vectors

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Introduction

- In this topic, we will
 - Revisit 2-dimensional vectors
 - Revisit 3-dimensional vectors
 - Introduce n -dimensional vectors
 - Describe representations
 - Introduce the notation $\mathbf{R}^2$, $\mathbf{R}^3$ and $\mathbf{R}^n$





2-dimensional real vectors

- A 2-dimensional real vector is an ordered pair of real numbers

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

- The vector is represented by a boldface roman lower-case letter
- The entries u_1 and u_2 are the corresponding italicized letters
 - These are assumed to be real numbers
- Examples:

$$\mathbf{u} = \begin{pmatrix} 3.72 \\ -2.54 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} 8.29 \\ 1.52 \end{pmatrix}$$

$$\begin{pmatrix} 4.38 \\ -13.54 \end{pmatrix}$$





2-dimensional real vectors

- Note that the order is relevant:
 - The vectors

$$\begin{pmatrix} 3.72 \\ -2.54 \end{pmatrix} \quad \begin{pmatrix} -2.54 \\ 3.72 \end{pmatrix}$$

are two distinct 2-dimensional real vectors





2-dimensional real vectors

- Recall that $\mathbf{R}$ represents the set of all real numbers
- We represent the set of all 2-dimensional real vectors with $\mathbf{R}^2$
- If $\mathbf{u}$ is a 2-dimensional vector,
we can write $\mathbf{u} \in \mathbf{R}^2$, which is read “ $\mathbf{u}$ is in the set $\mathbf{R}$ squared”
 - Alternatively, “ $\mathbf{u}$ is a 2-dimensional real vector”
- Note that $\mathbf{u} = \begin{pmatrix} 3.72 \\ \infty \end{pmatrix}$ is not in $\mathbf{R}^2$ because infinity is not a real number





3-dimensional real vectors

- A 3-dimensional real vector is an ordered triplet of real numbers

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

- The entries u_1 , u_2 and u_3 are assumed to be real
 - If the vector is written as $\mathbf{w}$, the three entries are w_1 , w_2 and w_3
- Examples:

$$\mathbf{u} = \begin{pmatrix} 5.92 \\ 0.00426 \\ 8320. \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} 52.3 \\ 9.5 \\ 17.8 \end{pmatrix}$$





3-dimensional real vectors

- We represent the set of all 3-dimensional real vectors with
 $\mathbf{R}^3$
- If $\mathbf{v}$ is a 3-dimensional vector,
we can write $\mathbf{v} \in \mathbf{R}^3$, which is read “ $\mathbf{v}$ is in the set $\mathbf{R}$ cubed”
 - Alternatively, “ $\mathbf{v}$ is a 3-dimensional real vector”





n -dimensional real vectors

- If n is a positive integer,
a real n -dimensional vector is an ordered sequence of n real numbers written as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{n-1} \\ u_n \end{pmatrix}$$

where each u_1 through u_n are real numbers





n -dimensional real vectors

- We will represent all n -dimensional real vectors by

$$\mathbf{R}^n$$

- You can read $\mathbf{w} \in \mathbf{R}^n$ as “ $\mathbf{w}$ is an element of $\mathbf{R}$ to the n ”
 - Alternatively, “ $\mathbf{w}$ is an n -dimensional real vector.”

- For example, $\begin{pmatrix} 1.2 \\ -3.7 \\ 6.4 \\ 0.8 \\ 9.5 \end{pmatrix} \in \mathbf{R}^5$ and $\begin{pmatrix} 3.0 \\ 5.4 \\ 0.9 \\ -1.5 \\ 2.4 \\ -1.8 \\ 0.7 \\ -0.5 \end{pmatrix} \in \mathbf{R}^8$





Sequences of vectors

- If we have four vectors in $\mathbf{R}^5$, we may label those vectors

$$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$$

- The entries of the vector $\mathbf{v}_k$ will be the real numbers

$$v_{k,1}, v_{k,2}, v_{k,3}, v_{k,4}, v_{k,5}$$

$$\text{or } \mathbf{v}_k = \begin{pmatrix} v_{k,1} \\ v_{k,2} \\ v_{k,3} \\ v_{k,4} \\ v_{k,5} \end{pmatrix} \in \mathbf{R}^5 \text{ for each } k = 1, 2, 3, 4$$





Handwritten vectors

- If a vector $\mathbf{v}$ is hand-written,
it will be shown with an arrow above the letter, and
the entries will not be italicized

- For example, $\vec{\mathbf{v}} = \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \\ \mathbf{v}_4 \\ \mathbf{v}_5 \end{pmatrix}$





Summary

- Following this topic, you now
 - Know that an n -dimensional real vector has n ordered values
 - Know the set of all n -dimensional vectors is represented as $\mathbf{R}^n$
 - Understand if $\mathbf{v}$ is a vector, then the entries are v_1 through v_n
 - May be frustrated that vectors are represented as columns, but the absolute necessity of this will become obvious





References

- [1] [https://en.wikipedia.org/wiki/Vector_\(mathematics_and_physics\)](https://en.wikipedia.org/wiki/Vector_(mathematics_and_physics))





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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